Inequality in a Two-agent variety extension growth model

Nyemwererai Matshaka

Abstract

This paper analyses how innovation, through a mechanism of Research and Development productivity, affects growth and consumption inequality. We develop a model that combines the two agent, worker-capitalist framework with a R&D based endogenous growth model and further extend to include taxes and subsidy as policy intervention. We find that increased innovation aids growth through improved productivity in R&D. We establish a consumption gap index based on the marginal propensities of consumption. The model shows that innovation through R&D decreases inequality in consumption. R&D productivity increases product accessibility and economic wealth hence increasing the redistribution potential. Quantitatively we analyse the extended economy in which we include a subsidy and tax on income. Focusing on the South African economy we find a negative relationship between innovation, through R&D productivity, and consumption inequality. Tax policy and a R&D investment subsidy reduces the gap in income and hence consumption which arises from increased innovation showing the role of policy.

Key Words: *economic growth; consumption inequality; R&D productivity; product variety*

JEL CLASSIFICATIONS D31, E21, E25, O11, O31, O40

1 Introduction

Despite different global trends in equality it remains a challenge while innovation is on the rise globally (The World Social Report 2020). This motivates researchers to study their relationship given that both impact policy and growth. The question of whether innovation deters redistribution remains standing in macroeconomics. Innovation and inequality relate in different ways, dependent on the type of inequality and the economic agents affected. In recent times there is an evolving understanding of the importance of heterogeneity of economies and their agents. This study considers the effect of innovation, through Research and Development, on growth and consumption inequality in a heterogeneous household economy and the role of redistributive fiscal policy.

Inequality is a crucial problem recognised world wide, posing a social challenges that economists seek to address particularly in the face of global mega-trends such as innovation ad technological breakthroughs. Innovation and technology over recent decades continues to expand with growth studies identifying the importance of knowledge accumulation. However with this expansion exists skepticism with regards to the long run effects on macroeconomic outcomes. Issues of redistribution are not exempt from these concerns. It is acknowledged that innovation aids growth and expands economic opportunities. This increases economic wealth and the potential for redistribution which may translate to reduced inequality. It is however also postulated that innovation may fuel inequality given issues of limited accessibility. Decisions and benefits of innovation are concentrated amongst the rich who are able to afford a wider variety of products from extended innovation. This therefore associates R&D with increased inequality rather than improved redistribution. The study is extended to analyse the role of policy in the relationship between innovation and inequality by introducing tax on income and a subsidy on R&D investment.

We adopt an endogenous Romer variety growth model (Romer, 1990) which we extended by modelling a two agent household framework to introduce heterogeneity.Following Debortoli and Galí (2017) and Broer et al. (2020) the two-agent model is characterised by workers and capitalist who differ in income and access to the asset market. This highlights the notion of extreme concentration in equity ownership. Workers are hand to mouth consumers while capitalists participate in the asset market and are able to smooth out lifetime consumption. Final goods production utilises intermediate goods whose variety is extended through creation of blueprints from R&D efforts. Capitalists own the monopolistically competitive intermediate good producing firms and earn profit.

The model shows that there is consumption inequality owing to differences in income. Innovation, through a mechanism of R&D productivity, has a negative distributive effect with regards to consumption inequality .The intuition is that innovation increases variety and therefore accessibility allowing for consumption at different income levels. Innovation also aids in increasing economic growth increasing the potential for growth to translate to reduction of inequality. Analytically we confirm that in the long run growth is driven by innovation in the economy. Policy intervention through the inclusion of tax on asset and labour income as well as a subsidy on R&D investment highlight that policy aids in the redistribution of wealth as innovation decreases consumption inequality by a larger magnitude.

Quantitatively we analyse the dynamics of capitalist consumption and product variety in the presence of policy intervention, calibrating for the South African economy which is characterised by high inequality as wealth is concentration among a small proportion of the population. We find that innovation through R&D productivity is negatively related to consumption inequality and positively with economic growth. This concurs with the analytical findings. Tax on labour income and the subsidy on R&D are negatively associated with consumption inequality. However asset income tax is found to have a close to zero positive impact on inequality. These findings highlight the importance of how policy addressing innovation for growth should consider the trade off between being both pro-growth and redistributionary.

The novel two -agent model related to this study is gaining traction in macroeconomics especially in New Keynesian frameworks commonly used in policy analysis. A number of these studies Broer et al. (2020); Walsh (2016); Bilbiie (2018) have focused on pricing and monetary policy heterogeneity. We follow studies of two-agent households with distinct group differences with regards to income versus the traditional model that is characterised by constrained versus unconstrained groups of agents with regards to specified market participation (Cantore and Freund (2021); Debortoli and Galí (2017)). This framework seeks to address the failure of the traditional two agent model in analysis.[1] This study complements studies using this novel model to contribute to growth-inequality literature by applying it to an endogenous R&D growth model, acknowledging the importance of accounting for agent heterogeneity with the purpose of replicating real world economies.

A strand of literature related to the current study concerns how growth relates to R&D and inequality. Various channels have been suggested to explain the growth-inequality relationship

[1]The traditional two agent model is criticised for failure to obtain consumption dynamics consistent with stylised facts of micro data and demand shock effects are based on profit income effects on labour supply which are less than convincing.

stretching from socio-political economy (Benhabib & Rustichini, 1996), to credit market imperfections (Benabou, 1996, 2002) and more recently fertility (De La Croix & Doepke, 2003). The literature remains inconclusive. Studies on growth have historically focused on physical and human capital as growth determinants. However the role of R&D as an important growth driver is now widely acknowledged (Sylwester, 2001). Chu et al. (2015) and Chu and Cozzi (2014) analyse the effect of monetary policy on growth and welfare in a Schumpeterian framework with representative households where a cash in advance restriction is placed on consumption, R&D and manufacturing investment by setting a cash requirement in order to aid R&D. In our study households are characterised by individuals who are differentiated by asset ownership as only a fraction of individuals own firms. Therefore we capture heterogeneity in the economy by diverging from the assumption that all households own firms.

We build on the growth, R&D and inequality literature such as Arawatari et al. (2018), Chu and Cozzi (2018),Chu et al. (2017, 2019), Zheng et al. (2020). These studies have applied R&D growth models to investigations that seek to understand how inequality and growth relate to various macroeconomic aspects such as monetary policy. The process of product innovation is highlighted to play an important role in determining how inflation affects inequality based on the intuition that variety disproportionately benefits higher income earners and therefore extends the inequality gap. Heterogeneity is also captured by modelling households with different asset holding. Recently Chan et al. (2022) study how R&D policy tools affect income inequality in an endogenous growth model with household heterogeneity. As a compliment, our study utilises the aspect of household heterogeneity in an endogenous growth model to investigate how innovation relates to inequality through the evaluation of marginal propensities to consume.

The rest of this paper is organised such that in Sections 2 and 3 we outline the two-agent endogenous growth model and analyse analytically and numerically in Sections 4 and 5. Section 6 concludes.

2 Model

In discrete time we set up a two agent household R&D growth model in the spirit of Romer (1990). The household in the two-agent model is characterised by two types of heterogeneous agents who are either workers or capitalists (Broer et al. (2020);Debortoli and Galí (2017)). This is referred to as a two-agent worker-capitalist model. Agents are ex ante identical with exception of asset ownership by capitalists who own firms and earn from returns on capital.

Workers are however rewarded solely from their labour. This model highlights the concentration of equity ownership within only a specific group and therefore introducing income inequality. A single final good is produced by a representative producer using labour and intermediate goods and intermediate goods are produced by monopolistically competitive firms which produce N_t number of differentiated goods in period t. The expansion of these differentiated products, through R&D, drives economic growth as in Romer (1990).

2.1 Households

In the two-agent model, heterogeneity is introduced by assuming that there are two households whose accessibility to assets differ (Debortoli & Galí, 2017); (Broer et al., 2020).
The household utility, as in (Broer et al., 2020), is given by

$$U = \sum_{t=0}^{\infty} \beta^t \big(\ln c_t - \frac{l_t^{1+\psi}}{1+\psi} - \nu \big) \tag{1}$$

where c_t is consumption level, l_t is labour supplied by households for a wage w_t and ν is a fixed cost of labour provision which discourages capitalists from working.

2.2 Capitalist

In the economy a continuum of households are represented in a unit interval and a constant fraction, λ, that chose not to work,are referred to as capitalists. These households own intermediate goods firms and invest capital into the production of these goods. Households maximise utility subject to unconstrained asset accumulation given by

$$c_t^c + a_{t+1} = (1 + r_t)a_t \tag{2}$$

where a_t denotes the assets in the economy comprised of capital, k_t, and the value of innovative firms owned. The rate of return on assets net depreciation is r_t. Standard dynamic optimisation yields

$$\frac{c_{t+1}^c}{c_t^c} = \beta(1 + r_{t+1}) \tag{3}$$

as the intertemporal optimality condition showing the trade off between current consumption and future gains from capital investment for the capitalist.

2.3 Worker household

The remaining $(1-\lambda)$ agents in the economy are constrained as they do not participate in the asset market. They therefore strictly consume the income earned from supplying labour. Utility is similar to that of the capitalist subject to their period budget constraint

$$c_t^w = w_t l_t \tag{4}$$

where l_t is labour supplied by households for a wage, w_t, determined by a labour union as follows

$$w_t = l_t^{\psi} c_t^w \tag{5}$$

where ψ is the Frisch elasticity of labour. We assume constant labour supply which ascertains hand-to-mouth consumption.

3 Firms

In this section we outline the production sector utilising a Romer type of R&D growth model closely following Aghion and Howitt (2008) and Chu (2021). The sector is characterised by final good production and intermediate good production, under which R&D also takes place.

Final goods firm

The market is perfectly competitive and we assume a representative final goods firm. Production requires a continuum of intermediate goods produced by capitalists with firms indexed by j and the labour allocated to final good firms. The production technology is

$$y_t = l_t^{1-\alpha} \sum_{j=1}^{N_t} {x_{jt}}^{\alpha} \tag{6}$$

where N_t is the number of differentiated intermediate goods in period t and x_{jt} is the quantity of intermediate input from firm j utilised in period t. Labour, $l_t = \sum_0^1 (l_{ijt})$ is demanded by firm j in order to produce final goods y_t. The inverse elasticity of substitution between inputs is $\alpha \in (0, 1)$.

The final goods firm maximises profit by choosing the amount of labour, l_t, and intermediate input, x_{jt}. The optimal labour for producing the final good is determined by the wage . Profit maximisation yields optimal labour

$$w_t = (1 - \alpha) \left(\frac{x_{jt}}{l_t} \right)^{\alpha} N_t \tag{7}$$

and optimal quantity of differentiated goods, x_{jt}

$$p_{jt} = \alpha \left(\frac{l_t}{x_{jt}} \right)^{1-\alpha} \tag{8}$$

where p_{jt} and w_t are intermediate good price and wage respectively.

Intermediate goods firm

There is a unit continuum of firms, indexed by j, owned by capitalists producing differentiated products in a monopolistic competitive market. Production of the differentiated goods requires knowledge in order to achieve variety extension. Monopolists rent capital as a production input at an one-to-one. The production technology is

$$x_{jt} = k_{jt} \tag{8a}$$

where $k_t = \sum_0^1 k_{jt}$ such that one unit of capital is needed to produce a single intermediate good.

The intermediate goods producer aims to maximise profit subject to the intermediate goods demand function given by (8). The marginal cost of producing an intermediate good is the rental rate cost of capital, $R_t = r_t + \delta$ where δ is the depreciation rate, and hence total cost is equal to $(R_t)x_{jt}$. The firm chooses the price, p_{jt}, of differentiated goods by maximising profit,

$$\max_{p_{jt}} \Pi_{jt} = p_{jt} x_{it} - R_t x_{jt} \tag{9}$$

Substituting for p_{jt} using (8) and maximising yields

$$p_{jt} = \frac{R_t}{\alpha} \tag{10}$$

as the optimal price of intermediate goods. Substituting for price in the demand function (8) gives the optimal quantity of differentiated goods to therefore supply. Due to price symmetry, given a arbitrage free condition, quantity becomes $x_j = x_{jt} \forall t$, such that output of intermediate good firms j is symmetric across firms.The optimal quantity produced is therefore

$$x_t = \left(\frac{\alpha^2}{R_t}\right)^{\frac{1}{1-\alpha}} l_t \tag{11}$$

which is related to the inverse price of capital. Substituting into (9) for p_{jt} and x_t using, (8a), (10) and (11) and given the symmetry condition and that $x_{jt} = x = \frac{k_t}{N_t}$, the optimal profit for intermediate goods firms is

$$\Pi_t = R_t x_{jt} \left(\frac{1-\alpha}{\alpha}\right) \tag{12a}$$

$$\Pi_t = \alpha(1-\alpha)\left(\frac{k_t}{N_t}\right)^{\alpha} l_t^{1-\alpha} \tag{12b}$$

Intermediate good firms profit is also symmetric given the constant labour allocated towards final goods production.

3.1 Research and Development

Research is carried out to create a blueprint for product extension, in order to facilitate growth. Cost of entry is financed by the intermediate goods producing firms' profit. A free entry condition requires that the present value of profits earned from blueprints is equal to the cost of creation.

$$v_t \phi F_t = F_t$$

The market is competitive such that the zero profit condition yields

$$v_t = \frac{1}{\phi} \tag{13}$$

where ϕ is the positive productivity parameter for the research department, F_t are units of final goods allocated to R&D and v_t is the value of each blueprint . This highlights the opportunity cost of holding other assets or more equity versus creating a blueprint. Product variety is measured by N_t which is the number of differentiated products in period t. The production function in R&D shows the dynamics of differentiated goods:

$$\Delta N_{t+1} = N_{t+1} - N_t = \phi F_t \tag{14}$$

The growth of product variety depends on the productivity within the sector, ϕ. The no arbitrage condition that determines the value of blueprints is

$$v_t = \frac{\Pi_t + v_{t+1}}{1 + R_t}$$

which is the present value of the profit flow of R&D firms.

$$R_t = \frac{\Pi_t}{v_t} + g_v$$

The interest rate is the sum of monopolistic profits and capital gain where g_v is the growth rate of v_t which is zero as, $v_t = \frac{1}{\phi}$, is constant. Therefore the value of each blueprint to its owner is given by (15).

$$v_t = \frac{\Pi}{R_t} \tag{15}$$

Substituting for v_t using (13) yields the equilibrium interest rate

$$R_t = \Pi\phi \tag{16}$$

R_t is equal to profit that are generated from investing into R&D and in order to produce intermediate goods. This allows for research arbitrage. Substituting for Π_t using (12b) and noting that $r_t = R_t - \delta$ yields

$$r = \alpha^2 \left(\frac{\phi(1-\alpha)}{\alpha} \right)^{(1-\alpha)} - \delta \tag{17}$$

showing that the rate of return is constant. Therefore the profit in intermediate good production in (12b) is both symmetric and constant over time.

4 Decentralised Equilibrium

We define the economy in equilibrium which outlines the path of allocations $\{c_t, l_t, y_t, x_{jt}, \Pi_t\}$ and their prices $\{p_{jt}, r_t, w_t, R_t, v_t\}$ such that in each period

- households maximise utility taking $\{r_t, w_t\}$ as given
- capitalist households hold capital and rent it out to intermediate good firms at a rental price $\{R_t\}$
- worker households provide labour $\{l_t\}$ and consume per period income $\{w_t\}$
- final goods $\{y_t\}$ are produced using $\{l_t\}$ and $\{x_{jt}\}$ in a perfectly competitive market and the firm seeks to maximise profit taking $\{w_t\}$ as given
- monopolistic producers make N_t variety of intermediate goods $\{x_{jt}\}$ using $\{k_t\}$ and maximise profit by choosing $\{p_{jt}\}$ while taking $\{R_t\}$ as given
- R&D is carried out in a competitive market and firms choose final good investment $\{F_t\}$ for maximisation of expected profit, taking value of blueprints $\{v_t\}$ as given
- the market clearing condition for final goods holds when $\{y_t = c_t + F_t + k_{t+1} - (1-\delta)k_t\}$
- the market clearing condition for labour is $\{\sum_0^1 l_{it} = l_t = 1\}$
- the total value of assets is a combination of capital stock and value of firms owned $\{k_t + v_t N_t\}$

4.1 Equilibrium Dynamics and Aggregate Economy

The aggregate economy is defined following the above outlined equilibrium conditions. This section outlines total output, consumption, intermediate goods, assets and economy wide resources.

Aggregate Output

Aggregate output is obtained by substituting for $x_{jt} = x = \dfrac{k_t}{N_t}$ in (6) and noting that equilibrium labour $l_t = l = 1$ such that

$$y_t = N_t^{1-\alpha} k_t^{\alpha} \tag{18}$$

capital and intermediate goods are combined to produce total output.

Aggregate Consumption

Aggregate consumption in the economy is defined as the sum of worker and capitalist household consumption

$$c_t = (1-\lambda)c_t^w + \lambda c_t^c$$

where

$$c_t^w = w_t = (1-\alpha)k_t^\alpha N_t^{1-\alpha} \tag{19a}$$

is the equilibrium the workers consumption expressed as a function of product variety, N_t and capital.

The capitalist consumption is obtained from the capitalist households optimal condition

$$c_t^c = \big(\beta(1+r)\big)^{-1} c_{t+1}^c$$

which, using iteration, simplifies to

$$c_t^c = \bigg(\beta(1+r)\bigg)^t c_0^c \tag{19b}$$

such that aggregate consumption is

$$c_t = (1-\lambda)(1-\alpha)k_t^\alpha N_t^{1-\alpha} + \lambda[(1+r)\beta]^t c_0^c \tag{20}$$

Total consumption in the economy for all household types is shown as a function of income.

Aggregate differentiated goods

The product variety dynamics show the extension of differentiated goods over time which drives economic growth. Using (14) we outline the units of final goods used as input in R&D

$$F_t = \frac{N_{t+1} - N_t}{\phi} \tag{21}$$

stating that the change in number of differentiated goods is determined by productivity and units of final goods devoted to R&D. Investment into R&D is positively asscoiated with number of differentiated goods and inversely with productivity.

Assets

Total assets comprise of capital and value of firm ownership. The aggregate asset equation is

$$\begin{aligned} A_t \equiv \lambda a_t &= k_t + v_t N_t \\ &= k_t + \frac{N_t}{\phi} \end{aligned} \tag{22}$$

where v_t is substituted for using (13).
Assets are owned only by capitalists therefore the aggregate capitalist household budget constraint is

$$\lambda(c_t^c + a_{t+1}) = \lambda(1 + r_t)a_t \tag{23}$$

and substituting for λa_{t+1} and λa_t using (22) gives

$$\lambda c_t^c = (1 + r)\left(\frac{\phi k_t + N_t}{\phi}\right) - \left(\frac{\phi k_{t+1} + N_{t+1}}{\phi}\right) \tag{24}$$

Aggregate capitalist consumption is a function of the dynamics of capital stock and product variety.

Aggregate Economy Resource Constraint

The economy wide resource constraint is given by

$$y_t + (1 - \delta)k_t = c_t + k_{t+1} + F_t \tag{25}$$

which outlines that resources y_t and k_t in the economy are utilised for consumption,intermediate goods production and investment into R&D. We substitute for y_t, c_t and F_t using (18), (20) and (21) respectively and note that $k_t = N_t x$. The economy wide resource constraint is therefore

$$\frac{N_{t+1}}{\phi} - k_{t+1} = [1 - (1 - \lambda)(1 - \alpha)]N_t^{(1-\alpha)} k_t^{\alpha} - (1 - \delta)k_t - \frac{N_t}{\phi} - [(1 + r)\beta]^t c_0 \tag{26}$$

The dynamics system of the economy constitutes of two equations in terms of capitalist consumption c_t^c and product variety N_t. Utilising (3) and substituting for $r_t = \alpha^2 k_t^{\alpha-1} - \delta$ while noting that $k_t = N_t x$, we obtain the capitalist consumption dynamics as

$$c_{t+1}^c = \beta(1 + \alpha(N_t x)^{\alpha-1} - \delta)c_t^c \tag{27}$$

We derive the dynamics of N_t from assets owned in (24) such that

$$(x + \frac{1}{\phi})N_{t+1} = \Big([\alpha^2(N_t x)^{\alpha-1} - \delta](x + \frac{1}{\phi}) + 1\Big)N_t - \lambda c_t^c \tag{28}$$

The transitional dynamics of the economy are driven by capitalist consumption c_t^c and product variety N_t as described by system (27) and (28).

4.2 Balanced Growth Equilibrium

The balanced growth path (BGP) is defined as one on which all real variables in the economy grow at the same constant rate.

Lemma 1: *The economy jumps to a steady balanced growth path with constant and symmetric long run growth for all real variables*

Proof. in Appendix A.

In order to ascertain that growth in capitalist consumption is along the BGP we utilise the economy's resource constraint in (25). We substitute for c_t^w and F_t using (19a) and (21) and take into consideration that $y_t = \frac{1}{\alpha^2 R} N_t x$ from (6) and (11) and $k_t = N_t x$ to obtain

$$N_{t+1} = \left\{\left[1 + \left(\frac{1-(1-\alpha)(1-\lambda)}{\alpha^2 R} + (1-\delta)\right)x + \frac{1}{\phi}\right]N_t - \phi\lambda c_t^c\right\}\left(x + \frac{1}{\phi}\right)^{-1} \tag{29}$$

a difference equation that the number of intermediate goods, N_t, satisfies.

Proposition 1: *Capitalist consumption is on the balanced growth path*
Proof. *in Appendix B and text*

Solving for the difference equation as shown in Appendix B we find the behaviour of N_t as

$$N_t = \left(N_0 - \frac{\phi\lambda c_0}{Q-(1-g_{c^c})}\right)Q^t + \frac{\phi\lambda c_0^c}{Q-(1-g_{c^c})}(1+g_{c^c})^t \tag{30}$$

where $Q_t = \left\{\left[1+\left(\frac{1-(1-\alpha)(1-\lambda)}{\alpha^2 R}+(1-\delta)\right)x+\frac{1}{\phi}\right]\left(x+\frac{1}{\phi}\right)^{-1}\right.$ and g_c^c is the growth in capitalist consumption.

The transversality condition from the capitalist household requires that $\lim_{t\to\infty}\beta^t c_t^{-1}a_{t+1}=0$ and as in Novales, Fernández, and Ruíz (2009), one of the conditions for the transversality condition to hold is

$$c_0 = \left(x+\frac{1}{\phi}\right)^{-1}\left[Q_t-(1+g_{c^c})\right]\frac{N_0}{\lambda} \tag{31}$$

Substituting for c_0 in (30) we obtain

$$N_t = N_0(1+g_{c^c})^t$$

and for $t=1$

$$g_N = g_{c^c} \tag{32}$$

Therefore showing that product variety and capitalist consumption grow at the same rate and from (A.0) and (A.1), $g_N = g_{c^c} = g_a = g_y = g_{c^w} = g_k = g_w$ which shows that the economy has a steady state. Growth is constant as given by

$$\begin{aligned} 1+g_{c^c} &= \beta(1+r) \\ g &= \beta\left(1+\alpha^2\left(\frac{\phi(1-\alpha)}{\alpha}\right)^{1-\alpha}-\delta\right)-1 \end{aligned} \tag{33}$$

This shows g as the balanced growth rate of the economy.

$$\frac{\delta g}{\delta\phi} = \beta\alpha^2(1-\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\phi^{-\alpha} > 0$$

for $0<\alpha,\beta<1$ and $\phi>0$.

Proposition 2: *Growth in the economy is driven by innovation through Research and Development*

Proof. Equation (33) shows that g is increasing in ϕ

Long run growth is shown to be a function of R&D productivity. Growth in the economy is positively related to R&D. Increase in R&D productivity ϕ is expected to yield greater innovation which leads to the positive relationship between growth and innovation.

4.3 Inequality in the Two agent household model

4.3.1 Consumption Inequality

The two households outlined in our paper, namely the capitalist and worker, have different sources of income which in turn determines their consumption level. Therefore we evaluate consumption disparities and the effect of innovation.

In order to analyse inequality we outline a consumption gap index as a ratio of steady state consumption between the two households. Inequality is denoted by $\Gamma \equiv 1 - \frac{c_t^c}{c_t^w}$ such that when $\frac{c_t^c}{c_t^w}$ is greater (lesser) than 1, then inequality exists with capitalists consuming more(less) than workers $c_t^c > c_t^W$ ($c_t^c < c_t^w$)
Capitalist consumption, c_t^c is obtained from (24) and worker consumption is

$$(1-\lambda)c_t^w = (1-\lambda)w_t = (1-\lambda)(1-\alpha)\frac{N_t x}{\alpha^2 R}$$

The consumption gap between the groups of agents is

$$\Gamma = 1 - \frac{\alpha^2(r - g_N)(x\phi + 1)R}{(1-\alpha)(1-\lambda)x\phi} \tag{34}$$

where $R = r + \delta$; $r = \alpha^2 \left(\frac{\phi(1-\alpha)}{\alpha}\right)^{1-\alpha} - \delta$ and $x = \frac{\alpha}{(1-\alpha)\phi}$

The consumption gap shows differences in marginal propensities to consume between agents which gives rise to disparities between agents. Inequality is a function the R&D productivity, (ϕ), capital intensity in production, (α), growth in innovation, g_N and rate of return, r.

Proposition 3: *The degree of consumption inequality is affected by the level of R&D productivity*

Proof. Equation (34) shows that Γ is decreasing in ϕ

$$\frac{\delta\Gamma}{\delta\phi} = -\frac{\alpha^3(1-\beta)\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\phi^{-\alpha}}{(1-\lambda)}\left[(1-\delta) + 2(\alpha^{1+\alpha}\phi^{1-\alpha}(1-\alpha)^{1-\alpha})\right] < 0 \tag{35}$$

which holds given that $0 < \delta, \alpha, \lambda, \beta < 1$ and noting that ϕ is a positive parameter.
Inequality in consumption is shown to be negatively related to the growth in innovation through a mechanism of R&D productivity, ϕ. This aligns with the intuition that increased R&D through increased productivity, ϕ, leads to increased variety of products which may lessen cost and increase accessibility of goods therefore reducing inequality in consumption.

4.4 Implications of fiscal policy

In this section we analyse the effect of policy on R&D driven growth and consumption inequality. An asset income and labour income tax is applied on the capitalist and worker household respectively and a subsidy for R&D is also introduced.

Capitalists and workers have the same utility given in (1) subject to $c_t^c + a_{t+1} = (1+r_t)a_t - \tau^a a_t r_t$ and $c_t^w l_t = w_t(1-\tau^l)$ respectively. The optimal condition for the capitalist becomes

$$\frac{c_{t+1}^c}{c_t^c} = \beta(1 + [1-\tau^a]r_t)$$

Using (22) and noting that asset income is taxed and $N_t x = k_t$ we obtain

$$\lambda c_t^c = (1 + (1-\tau^a)r_t)\left(x + \frac{1-s}{\phi}\right)N_t - \left(x + \frac{1-s}{\phi}\right)N_{t+1} \tag{36}$$

The aggregate budget of the worker who consumes strictly their after tax income is $c_t^w l_t^\psi = w_t(1-\tau^l)$ and the total worker consumption in the economy is

$$(1-\lambda)c_t^w = (1-\lambda)(1-\tau^l)w_t = (1-\alpha)(1-\lambda)(1-\tau^l)\frac{N_t x}{\alpha^2 R} \tag{37}$$

Government

Tax, denoted by τ^a and τ^l, is collected from capitalists on the assets and from workers on labour income respectively. The government uses this revenue to finance an R&D subsidy, s, such that the governments balanced budget is

$$\tau^a a_t r_t + \tau^l w_t l_t = sF_t \tag{38}$$

R&D with investment subsidy

The tax collected from households is used to subsidise the cost of R&D such that the free entry condition in R&D is

$$v_t \phi F_t = (1-s)F_t$$

and

$$v_t = \frac{(1-s)}{\phi}$$

substituting for v_t in (15) and noting that $R_t = r + \delta$ we get a constant rate of return as

$$r = \alpha^2 \left[\frac{\phi}{1-s}\left(\frac{1-\alpha}{\alpha}\right)\right]^{1-\alpha} - \delta \tag{39}$$

Growth effect with tax

The capitalist optimal condition, when tax is introduced, becomes

$$\frac{c_{t+1}}{c_t} = \beta(1 + [1-\tau^a]r)$$

and therefore growth along a balanced growth path is

$$g^\tau = \beta\left(1 + \left[\alpha^2\left(\frac{\phi(1-\alpha)}{(1-s)\alpha}\right)^{(1-\alpha)} - \delta\right](1-\tau^a)\right) - 1 \tag{40}$$

which is a function of asset income tax, R&D productivity and subsidy. Differentiating (40) with respect to asset income τ^a, R&D productivity, ϕ, and subsidy, s, yields

$$\frac{\delta g^\tau}{\delta \tau^a} = -\beta\left(\alpha^2\left(\frac{\phi(1-\alpha)}{\alpha(1-s)}\right)^{1-\alpha} - \delta\right) < 0 \tag{41a}$$

$$\frac{\delta g^{\tau}}{\delta \phi} = \beta(1+\tau^{a})\left(\alpha^{2}\left(\frac{(1-\alpha)}{\alpha(1-s)}\right)^{1-\alpha}(1-\alpha)\phi^{-\alpha} > 0 \tag{41b}$$

$$\frac{\delta g^{\tau}}{\delta s} = \beta(1+\tau^{a})\left(\alpha^{2}\left(\frac{\phi(1-\alpha)}{\alpha}\right)^{1-\alpha}(1-s)^{(\alpha-2)}(1-\alpha) > 0 \tag{41c}$$

Growth decreases by the level of tax imposed on asset income as highlighted by the negative relationship which holds given that all parameters are positive. Tax on asset holding lessens capital accumulation and discourages investment which in turn reduces growth. The subsidy is intuitively increases growth as it allows for increased investment in R&D. The positive relationship between growth and subsidy holds for $(1-s)^{\alpha-2} > 0$ given that $0 < s < 1$. Growth remains innovation driven given that growth is a function R&D productivity ϕ and R&D policy, through a subsidy s.

Consumption inequality implication with a tax

The consumption inequality index is given by $\Gamma_{\tau} = 1 - \frac{c_t^{c\tau}}{c_t^{w\tau}}$ which holds for similar conditions for the ratio of each household consumption as before the introduction of tax. Utilising (36) and (37 we obtain the consumption inequality index as

$$\Gamma_{\tau} = 1 - \frac{\Big\{(1-\tau^{a})r - g_{N}^{\tau}\Big\}\alpha^{2}R\big(x\phi + (1-s)\big)}{(1-\lambda)(1-\alpha)(1-\tau^{l})x\phi} \tag{42}$$

where $R = r + \delta$ and $x = \frac{\alpha(1-s)}{\phi(1-\alpha)}$. Consumption inequality is a similar function to (34) but extends to a function of taxes on inequality and the R&D subsidy. Therefore consumption inequality is affected by innovation through a mechanism of productivity, ϕ, the subsidy, s and the rate of labour and asset income tax.

Differentiating (42) with respect to τ^{a} and τ^{l} as well as R&D productivity and subsidy, ϕ and s yields

$$\frac{\delta \Gamma_{\tau}}{\delta \tau^{a}} = \frac{\alpha^{3}\big(\phi\frac{1-\alpha}{\alpha}\big)^{1-\alpha}(1-s)^{\alpha-1}(1+\alpha)(1-\beta)}{(1-\alpha)(1-\lambda)(1-\tau^{l})}\left(\alpha^{2}\big(\frac{\phi(1-\alpha)}{\alpha(1-s)}\big)^{1-\alpha} - \delta\right) > 0 \tag{43a}$$

which holds noting that all parameters are positive and that $0 < s < 1$ such that $(1-s)^{\alpha-1} > 0$. Inequality is therefore increasing in tax on asset income.

$$\frac{\delta\Gamma_\tau}{\delta\tau^l} = \frac{-1}{(1-\tau_l)^2}\frac{\Big\{(1-\tau^a)r - g_N^\tau\Big\}\alpha^2 R\big(x\phi + (1-s)\big)}{(1-\alpha)(1-\lambda)} < 0 \tag{43b}$$

This highlights a negative relationship between labour income tax and consumption inequality which holds given that all parameters are positive.

$$\frac{\delta\Gamma_\tau}{\delta s} = \frac{\alpha^3\big(\frac{\phi(1-\alpha)}{\alpha}\big)^{1-\alpha}(1+\alpha)}{(1-\alpha)(1-\tau^l)(1-\lambda)}(1-\beta)(1-s)^{\alpha-2}(\alpha-1) < 0 \tag{43c}$$

Increasing the R&D subsidy decreases inequality in consumption. This condition holds for all positive parameters and that $0 < \alpha < 1$ such that $(\alpha - 1) < 0$

$$\frac{\delta\Gamma_\tau}{\delta\phi} = \frac{\alpha^3(\frac{1-\alpha}{\alpha})^{1-\alpha}(1+\alpha)}{(1-\alpha)(1-\tau^l)(1-\lambda)}\phi^{-\alpha}(\alpha-1) < 0 \tag{43d}$$

A negative relationship exists between inequality and R&D productivity when tax is introduced in the economy which holds for $0 < \alpha < 1$. Increase in innovation through increased R&D productivity, ϕ, leads to a decrease in consumption inequality. This result corresponds to that in the initially discussed economy without policy intervention.

5 Numerical Analysis

To further analyse the interaction between consumption inequality and tax policy as the impact of R&D, we calibrate the model and carry out a numerical analysis. We calibrate a benchmark economy with standard parameters that seek to reflect the South African economy.

5.1 Calibration

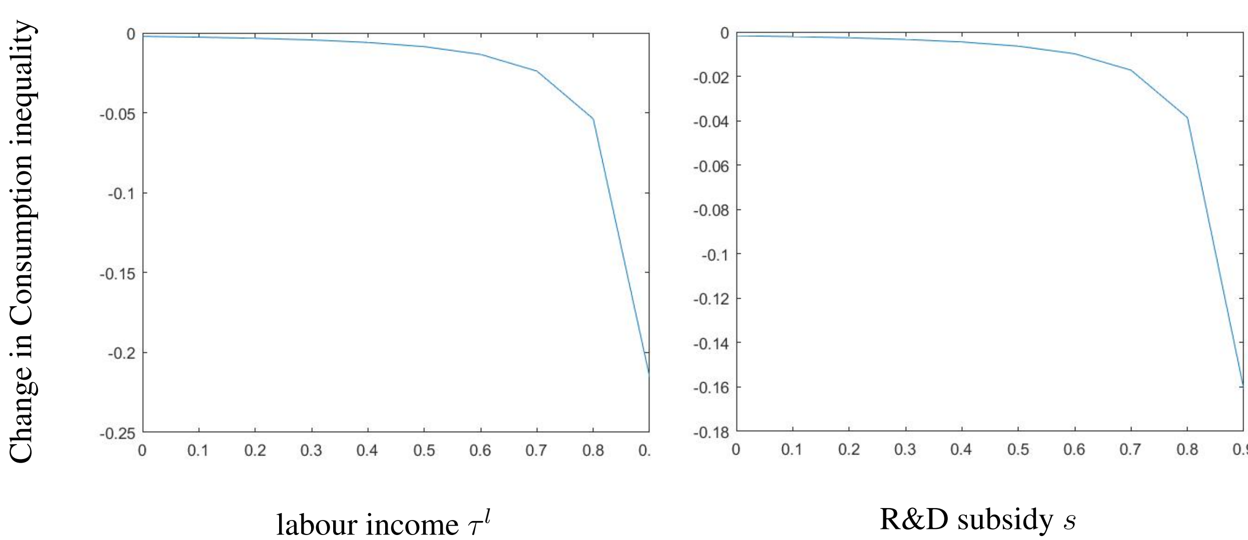
Table 1 outlines the benchmark parameters of the economy. We set the discount factor $\beta = 0.96$ as in Getachew and Turnovsky (2020), which is indicated to match a 4.17% rate of time preference for infinite lived agent models.

Table 1: Benchmark values

Baseline parameters			
Preference parameters	$\beta = 0.96$		
Production	$\alpha = 0.23$	$\delta = 0.1$	
Inequality	$\lambda = 0.10$		
Policy	$\tau^a = 0.168$	$\tau^l = 0.337$	$s = 0.16$

Production parameters include capital elasticity , $\alpha = 0.23$ obtained from du Plessis, Smit, Steinbach, et al. (2014), a study that focuses on South Africa. We obtain the capital depreciation value as $\delta = 0.1$ from Canales-Kriljenko (2011). In order to carry out the sensitivity test we experiment for different values of R&D productivity, ϕ. Increasing productivity translates to improved innovation. Capturing inequality we set the share of capitalists as $\lambda = 0.1$. In South Africa only 10% of the population owns 80% of the nation's wealth (SA-TIED, 2020). According to the South African Revenue Service (SARS), the individual income tax for the 2022 tax year over different income groups ranges between $18 - 45\%$. The average income tax rate is 33.7%. We use the capital gains tax on individuals to measure asset income tax taking the average rate between $2015 - 2023$ at 16.84%. The subsidy is set at 0.16 based on the Implied Tax Subsidy Rates on R&D expenditures data obtained from the OECD.

Figure 1: Impact of policy on Consumption inequality



Labour income tax and the R&D subsidy are shown to have a consumption inequality reducing effect as shown in Fig 1. South Africa has a progressive income tax system and it is intuitive that higher taxes decrease inequality as they affect the wealthier, allowing policy to be redistributive. Although asset income tax, τ^a, is positively related to consumption inequality, the magnitude is small and therefore overshadowed by the negative effect of labour income tax.

$$\frac{\delta\Gamma_\tau}{\delta\tau^a} = 0.00012857 \tag{44}$$

We analyse the relationship between innovation and consumption inequality in the economies with and without tax by adjusting the level of R&D productivity.

Table 2: Consumption Inequality and Innovation

Consumption Inequality Effect of changes in R&D productivity

ϕ	$\frac{\delta\Gamma}{\delta\phi}$	$\frac{\delta\Gamma_\tau}{\delta\phi}$
0.5	-0.0026	-0.0030
1	-0.0029	-0.0033
1.5	-0.0032	-0.0037

The negative relationship between consumption inequality shown in Table 2 aligns with the analytical findings. Increasing R&D productivity and hence innovation, decreases inequality. We find that the effect of R&D productivity on consumption inequality is greater when a policy intervention is made through taxes on income and a subsidy. Capitalists, who own assets are taxed for increased income which arises from increased R&D productivity. This decreases the gap between the two households by reducing the increased income in one household type. Wealth in the South African economy is held by a small proportion of the economy which is outlined by capital in the model. Taxing this group reduces their income and therefore reduces the inequality gap. Innovation also drives growth increasing the potential of redistributive policies to spread the gains from innovation.

6 Conclusion

In light of the global breakthroughs and expansion in innovation and technology, curiosity breeds with regards to its role in the on going challenge of inequality reduction. It is of concern as the rapid growth of technology impacts existent policy that seeks to address the achievement of sustainable growth and living. This study therefore investigates the effect of R&D on growth in an economy with household income heterogeneity and its role in redistribution. We utilise an R&D based endogenous growth model in which R&D yields product variety with heterogeneous household. In the economy, household agents are either workers or capitalists and they differ in income source.

Increased innovation, through a mechanism of R&D productivity, drives economic growth. This matches prior studies of growth and R&D despite the introduction of heterogeneity in household incomes. The income disparities between households translate to differences in marginal propensities to consume, hence consumption inequality. We find that the gap in consumption between the two household types decreases given an increase in innovation through a mechanism of increased R&D productivity. This is explained by the increase in economic wealth which results from increased innovation which increases redistributive potential.

In order to further analyse the relationship while understanding the role of policy, we include a tax on income and a subsidy on R&D investment. We find that the inequality gap in consumption is lessened by innovation which stems from improved R&D productivity in the taxed economy. The calibration of the model confirms these findings for the South African economy. Policy intervention is also noted to increase the impact that innovation has on the reduction of inequality. These findings highlight the importance of considering heterogeneity in investigating macroeconomic relationships as this reflects real world economies. It also allows for R&D growth policy formulation process to aim to be pro-growth and inclusive. It is necessary that changes in technology not only benefit growth, but for policy to channel it towards the goal of reducing inequality.

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Appendix A

A Proof that all variables grow at the same constant rate

We outline the conditions for a balanced growth path beginning with aggregate output, (18) and noting that $x = \frac{k_t}{N_t}$ is constant. Aggregate capital grows at the same rate as product variety and therefore output also grows at the same rate

$$g_k = g_N = g_y \tag{A.0}$$

The total consumption of worker households

$$(1-\lambda)c_t^w = (1-\lambda)(1-\alpha)y_t$$

grows at the same rate as aggregate output such that

$$g_{c^w} = g_y \tag{A.1}$$

and from (23) we also note that

$$g_c^c = g_a \tag{A.2}$$

the assets grow at the same rate as capitalist consumption.

Appendix B

B Proof that capitalist consumption is on the Balanced growth path

We utilise the resource constraint (26) we substitute for c_t^w and F_t using (19a) and (21) and take into consideration that $y_t = \frac{1}{\alpha^2.R}N_t x$ and $k_t = N_t x$ to obtain

$$N_{t+1} = \left\{ \left[1 + \left(\frac{1-(1-\alpha)(1-\lambda)}{\alpha^2 R} + (1-\delta) \right) x + \frac{1}{\phi} \right] N_t - \phi\lambda c_t^c \right\} \left(x + \frac{1}{\phi} \right)^{-1} \tag{B.0}$$

which is a difference equation of product variety. Noting that $1 + g_c = (1+r)\beta$, iteration of $c_t^c = (1+r)\beta$ yields

$$c_t^c = (1+g_c)^t c_0^t \tag{B.1}$$

Substituting for c_t^c in the difference equation above yields

$$N_{t+1} = \left(\left\{ \left[\frac{1-(1-\lambda)(1-\alpha)+(1+\delta)}{\alpha^2 R} \right] x + \frac{1}{\phi} \right\} N_t - \lambda (1+g_c)^t c_0^t \right) \left(x + \frac{1}{\phi} \right)^{-1} \tag{B.2}$$

which has a particular solution $N_t = \beta(1+gc)^t$ and the homogeneous solution of the form $N_t = D\left(\left[\frac{1-(1-\lambda)(1-\alpha)+(1+\delta)}{\alpha^2 R} \right] x + \frac{1}{\phi} \right) \left(x + \frac{1}{\phi} \right)^{-1} \Bigg)^t$ for constants B and D.

Solving for B we substitute the proposed solution into B.2 such that

$$B = \frac{\lambda}{Q-(1+g_c)} c_0^c \left(x + \frac{1}{\phi} \right)^{-1}$$

where $Q_t = \left\{ \left[1 + \left(\frac{1-(1-\alpha)(1-\lambda)}{\alpha^2 R} + (1-\delta) \right) x + \frac{1}{\phi} \right] \left(x + \frac{1}{\phi} \right)^{-1} \right.$ The general solution for B.2 is

$$N_t = DQ^t + \lambda c_0^c \left(x + \frac{1}{\phi} \right)^{-1} \frac{1}{Q-(1+g_c)} (1+g_c)^t \tag{B.3}$$

At time $t = 0$

$$D = N_0 - \lambda \frac{c_0^c}{Q - (1 + g_c)} \left(x + \frac{1}{\phi} \right)$$

characterising the behaviour ofN_t, given B.3, as

$$N_t = \left(N_0 - \lambda \frac{c_0^c}{Q - (1 + g_c)} \left(x + \frac{1}{\phi} \right)^{-1} \right) Q^t + \lambda c_0^c \left(x + \frac{1}{\phi} \right) \frac{1}{Q - (1 + g_c)} (1 + g_c)^t \quad \text{(BB.4)}$$

Following (Novales et al., 2009), the transversality condition $\lim_{t=\infty} \beta^t c_t^{-1} a_{t+1} = 0$ holds for

$$c_0^c = N_0 \left(\frac{Q - (1 + g_c)}{\lambda} \right) \left(x + \frac{1}{\phi} \right)^{-1} \quad \text{(B.5)}$$

Substituting for c_0^c into BB.4 using B.5 yields

$$N_t = N_0 (1 + g_c))^t$$

where $R = r + \delta$ and $r = \alpha^2 \left(\frac{\phi(1 - \alpha)}{\alpha} \right)^{1-\alpha} - \delta$

We therefore yield a consumption gap of